PARTIAL OPENING OF A CRACK WITH INJECTION OF A VISCOUS LIQUID

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For a hydraulic breaking crack at a considerable depth the additional liquid pressure created in order to overcome rock strength is small compared with mine pressure P_g. Therefore the condition of smooth joining is approximately fulfilled [1, 2]. This condition is exactly correct in the stage of filling an already existing crack with liquid while it is only partly open, i.e., the joining lines for the sides do not reach the edge.

The smooth joining condition for a circular crack has the form [1, 2]

$$\lim_{X\to L}(L-X)^{-1/2}W(X)=0 \tag{1}$$

(X is radial coordinate in the crack plane, L is current smooth joining radius, W(X) is crack opening profile).

Below a smooth joining crack is studied assuming smoothness for the pumping regime when acceleration of the flow caused by a change in the overall flow rate of the injected liquid is assumed to be small [3].

We formulate the rest of the equations for the problem assuming that the material is elastic and the liquid injected from a point source into the center of the crack is incompressible and nonfiltering. We write the Sneddon equation [1] for the profile W(X) of a crack opening under the action of pressure P(X) in a more convenient form for the hydraulic breaking problem [4]

$$W(X)=-\frac{2}{\pi D}\int\limits_0^{\Gamma}P'(X_1)\left\{\sqrt{L^2-X^2}-\int\limits_{\max(X,X_1)}^{L}dX_2\left(\frac{X_2^2-X_1^2}{X_2^2-X^2}\right)^{1/2}\right\}dX_1. \tag{2}$$

Here Γ is loaded section radius; $\max(X, X_1)$ is the greater of the two numbers in brackets; $D = E[2(1-\nu^2)]^{-1}$ is a combination of standard elasticity constants; $P'(X)$ is radial pressure gradient. To the right-hand part it is necessary to add with a minus sign the value

$$W_g=\frac{2}{\pi D}P_g\sqrt{L^2-X^2}, \tag{3}$$

which takes account of the contribution of mine pressure [1].

Liquid flow in a narrow crack in an inertialess approximation is described by the Boussinesq equation [5]

$$P'=-3\mu Q(4\pi XW^3)^{-1},\ Q=Q_0-\partial\Omega/\partial T, \tag{4}$$

where μ is dynamic viscosity; $Q(X)$ is volumetric flow rate through a section of a crack having a radius X; Q_0 is source productivity, i.e., is the volume of liquid injected every second; $\Omega(X)$ is the volume of the central part of the crack cavity bounded by the section of fixed radius X referred to:

$$\Omega(X)=4\pi\int\limits_0^X X_1W(X_1)\,dX_1.$$

It is noted that $\Omega(\Gamma)$ agrees with volume Ω_s of liquid in the crack at the instant in question.

If we ignore the corrections caused by the rate of change in Q_0, then the mass conservation rule [second equation of (4)] may be transformed to

$$Q=Q_0q,\ q=1-\partial\Omega/\partial\Omega_s. \tag{5}$$

The set of equations is formulated. For simplification of analyzing it we change over to dimensionless variables. Instead of radial coordinates X and Γ we introduce the corresponding angular variables φ and γ by the equations $X = L\sin\varphi$, $\Gamma = L\sin\gamma$. As a radial scale we take $L_g = D(1.5\pi^2\mu Q_0)^{1/3}(2P_g)^{-4/3}$, and for the opening scale $W_* = (3\mu Q_0 L)^{1/4}(2\pi^2 D)^{-1/4}$, and we

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write an equation for the dimensionless change as follows:

$$W = W_* v \cos\varphi,\quad W_g = W_* v_g \cos\varphi,\quad L = L_g v_g^{4/3}.$$

Here v specifies dimensionless opening; v_g is joining radius. We also introduce characteristic volume $\Omega_g = 0.75\mu Q_0 (\pi D)^2 P_g^{-3}$ and with its help we make Ω and Ω_s dimensionless:

$$\Omega = \Omega_g v_g^3 \omega,\quad \omega = \int_0^\varphi v\theta d\psi,\quad \theta(\psi) = \sin\psi\cos^2\psi,\qquad \Omega_s = \Omega_g v_g^3 \omega_s,\quad \omega_s = \int_0^\gamma v\theta d\psi \tag{6}$$

(ω and ω_s are the corresponding dimensionless volumes).

We formulate the main equation determining $v(\varphi)$ in the filling zone $\varphi \leqslant \gamma$ (from what follows it will be seen that finding the rest of the dimensionless values is reduced to algebraic operations and quadratures). With this aim we substitute (4) in Eq. (2) written taking account of (3). By excluding P' and changing over to dimensionless variables, and also drawing attention to (1), after transformation we find that

$$v(\varphi) = Gf(\varphi) = \int_0^\gamma G(\varphi, \psi) f(\psi) d\psi,\quad f(\psi) = q(\psi)\theta^{-1}(\psi) v^{-3}(\psi),$$
$$G(\varphi, \psi) = \cos\psi - \cos^{-1}\varphi \int_{\max(\varphi,\psi)}^{\pi/2} d\eta \cos\eta \left(\frac{\sin^2\eta - \sin^2\psi}{\sin^2\eta - \sin^2\varphi}\right)^{1/2},\quad \varphi \leqslant \pi/2. \tag{7}$$

Condition (1), which in terms of $v(\varphi)$ has the form $v(\pi/2) = 0$, is satisfied automatically and the following representation is correct

$$v_g = g(f) \equiv \int_0^\gamma (1 - \cos\psi) f(\psi) d\psi. \tag{8}$$

In order to obtain from (7) an equation with respect to v it is necessary to express q in terms of v. In order to derive the appropriate relationship we substitute Eqs. (6) in (5). The scale multiple Ω_g depends on single variable Q_0 which is assumed to be a slowly changing function, and this means that it is possible to take Ω_g beyond the differentiation sign. By taking this into account after simple computations we have

$$q = 1 - \frac{3u_g\omega + v_g\omega_{,\gamma} - \frac{4}{3} u_g v \sin^2\varphi\cos\varphi}{3u_g\omega_s + v_g\omega_{s,\gamma}},$$
$$\omega_{,\gamma} = \int_0^\varphi u\theta d\psi,\quad \omega_{s,\gamma} = \int_0^\gamma u\theta d\psi + v(\gamma)\theta(\gamma). \tag{9}$$

Here $u = v_{,\gamma}$; $u_g = v_{g,\gamma}$; index γ after the comma means differentiation with respect to γ. Taking account of (9) relationship (7) in section $\varphi \leqslant \gamma$ is an equation with respect to the corresponding part of $v(\varphi)$, containing integral transformations with respect to variable φ and differentiation with respect to parameter γ. It would appear that a Cauchy problem arises with respect to γ. However, the actual dependence of q on u, u_g appears to be weak and it is easy to find an approximate expression for q free from differentiation with respect to γ. In order to substantiate this we draw attention to the fact that in the process of filling there are rapidly and slowly changing variables. In particular, radius L changes more rapidly than the degree of filling γ, and therefore the dependence on radius of the amount of opening appears to marked. According to the considerations of dimensionality W ~ L. In part, the dependence on L is contained in the scale multiple $W_* \sim L^{1/4} \sim v_g^{1/3}$, so that $v \sim L^{3/4} \sim v_g$. In view of this v may be presented in the form of a derivative of v_g and slowly changing function γ. If during differentiation derivatives from the second factor are ignored, then $u \simeq u_g v_g^{-1} v$. We substitute this equation in (9) and in subsequent numerical calculations we make the justified assumption of smallness of the value $0.25\, v_g v(\gamma)\theta(\gamma)(u_g\omega_s)^{-1}$ over the whole range of change in γ. Then for q we obtain an approximate expression which does not contain differential operations with respect to γ:

$$q \simeq q_0 = 1 - \omega_s^{-1}\left(\omega - \frac{1}{3} v \sin^2\varphi\cos\varphi\right). \tag{10}$$

Substitution of (10) in (7) gives a unidimensional nonlinear integral equation with respect to v. For convergence of subsequent approximations in the given class of integral

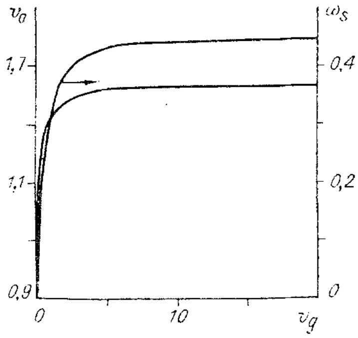

Fig. 1

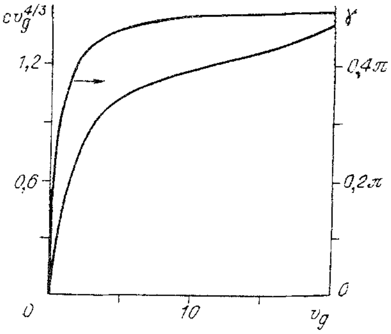

Fig. 2

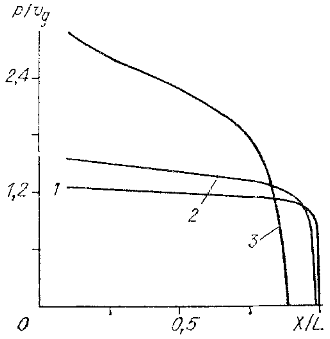

Fig. 3

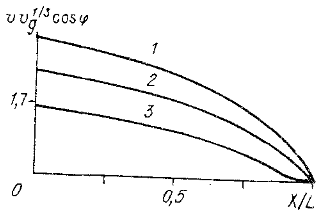

Fig. 4

equations with power nonlinearity it is necessary to transform the equation so that the right-hand part exhibits the property of zero level uniformity with action on $v(\varphi)$ [3, 4]. This condition is satisfied by the representation

$$v = G_0^{1/4},\quad G_0 = v^3(\varphi)\int_0^{\gamma} G(\varphi, \psi) f(\psi)\, d\psi,\quad f = q_0(\theta v^3)^{-1},$$

since it is evident that $G_0(\alpha v) = G_0(v)$ with α = const.

We note that Eq. (10) appears to be accurate for injection regimes of the form $Q_0 = \varkappa L^3$ ($\varkappa$ = const). Here γ = const, i.e., the corresponding solution is self-modeling [2].

It is possible to make approximation (10) more definite without turning to the Cauchy problem. With this aim we return from (10) to Eq. (9) but we shall consider v and u as independent variables and we attempt to obtain an additional equation for u not containing higher derivatives with respect to γ. By differentiating both parts of equalities (7) and (8) with respect to γ we have

$$u = -3G(fuv^{-1}) + G(q_{,\gamma}\theta^{-1}v^{-3}) + G_{,\gamma}f,\quad G_{,\gamma}f(\varphi) \equiv G(\varphi, \gamma)f(\gamma),$$
$$u_g = -3g(fuv^{-1}) + g(q_{,\gamma}\theta^{-1}v^{-3}) + g_{,\gamma}(f),\quad g_{,\gamma}(f) \equiv (1 - \cos\gamma)f(\gamma).$$

In order to be free from higher derivatives in these relationships, instead of $q_{,\gamma}$ we shall use an approximate expression

$$q_{,\gamma} \simeq q_{0,\gamma} = \omega_s^{-2}\left[\omega_{s,\gamma}\omega - \omega_s\omega_{,\gamma} - \frac{1}{3}(\omega_{s,\gamma}v - \omega_s u)\sin^2\varphi\cos\varphi\right], \tag{11}$$

and for convergence of the last approximations we rewrite them in the form

$$u = \frac{1}{4}[3(u - G(fuv^{-1})) + G(q_{0,\gamma}\theta^{-1}v^{-3}) + G_{,\gamma}f],$$
$$u_g = \frac{1}{4}[3(u_g - g(fuv^{-1})) + g(q_{0,\gamma}\theta^{-1}v^{-3}) + g_{,\gamma}(f)],$$

where $q_{0,\gamma}$ is taken from (11).

As numerical calculations showed, with use of approximation (10) the maximum error in determining v does not exceed 8% so that the first approximation is quite effective and a more precise definition of it is rather in principle of more practical value.

Now we consider how other variables are calculated after $v(\varphi)$ is determined in the filling zone. In the unwetted part v is found from Eq. (7), and v_g is found from (8). It is easy to show that pressure distribution is specified by a relationship of the form

$$P = P_g v_g^{-1} p, \quad p = \int_{\varphi}^{\gamma} f d\psi.$$

Dimensionless volume ω_s is calculated from (6). As a measure of yielding C for a crack we take the ratio of opening and pressure averaged for radius L. For C it is easy to obtain an expression

$$C = \frac{2L_g}{\pi D^{1/4}} v_g^{4/3} c, \quad c = \int_0^{\gamma} v \cos^2 \psi d\psi \Big/ \int_0^{\gamma} p \cos \psi d\psi.$$

If the joining radius L is previously unknown, then in accordance with the undimensional equations, in order to calculate dimensional quantities values of the following complexes are necessary: $v_g^{4/3}$ for radius, $v_g^{1/3} v \cos\varphi$ for opening, $v_g^3 \omega_s$ for the volume of liquid in the crack, p/v_g for pressure, and $v_g^{4/3} c$ for yielding. Presented in Figs. 1 and 2 are the dependences of $v_0 = v(0)$, ω_s, $v_g^{4/3} c$ and γ on parameter v_g. Profiles of pressure p/v_g and opening $v_g^{1/3} v \cos\varphi$ for some values of γ are shown in Figs. 3 and 4 (curves 1-3 for γ = 1.5, 1.4, 1.1, respectively). With an increase in v_g functions v_0, ω_s, p/v_g, and γ tend asymptotically towards a constant. This makes it possible to consider a crack of large size without having to resort to solving Eq. (7) with γ close to $\pi/2$.

Since variables are governed by a single parameter of state γ, in calculating dimensional values it is necessary to establish the connection between regime parameters of state at a given instant, scale multiples, and values of formal parameter γ. Let, for example, values of Q_0 and Ω_s be prescribed. Then Ω_g is calculated from Q_0 and the value of the complex $v_g^3 \omega_s = \Omega_s / \Omega_g$ is found. This makes it possible to determine parameter γ, and consequently the rest of the dimensionless complexes, after which it is easy to find the scale multiple and the corresponding dimensional values.

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